**Some Connection between Diffusion and IPR**

Consider the long time average probability of return to a particle’s initial *state*. Let the initial state be |ψ(0)>. Then it will evolve in time according to:



where |ψn> are the energy eigenfunctions. It’s time averaged probability of returning to its initial state will be:



In any event, in the large T limit, the phases will mostly cancel and we’ll just get the Em = En contribution, which is (note the denominator is canceled by the numerator when Em = En so we’re spared singularities) :



Can see the diffusion constant must be (inversely) related to this quantity, as it ought to be the case that the probability of return is super small for a diffusive particle. Now let’s say that our initial state is localized within some region. If *eigenstates* are localized, then our initial state will have relatively large overlap with just a few states (say |cm|2 = ½, ½ → Σ|cm|4 = ½), and so P(return)will be largish, close to 1. But if eigenstates are delocalized, then overlap will be with many states (need a lot of delocalized states to create a localized one – think FT), and so P(return) will be small (say |cm|2 = (1/10, 1/10, 1/10, …., 1/10) → Σ|cm|4 = 1/10).

Now let’s say that |ψ(0)> is exactly |**r**>. Then this becomes:



whereas the P-1 (the IPR) = ∫ddr |ψ(r)|4, for a given state. So we can say that the probability of return is equal to the sum of P-1 for the eigenfunctions.



If the eigenfunctions are localized, then this would be something of the ξ-d sort, and so finite. But if the eigenfunctions are all delocalized (or even if one is), then it’ll be like L-d, and so 0.